THURSDAY, APRIL 26, 1883

SCIENTIFIC WORTHIES

XXI.—WILLIAM SPOTTISWOODE

WILLIAM SPOTTISWOODE, President of the Royal Society, was born in London, Jan. 11, 1825. He belongs to an ancient Scottish family, many members of which have risen to distinction in Scotland and also in the New World.[1] He was first sent to a private (we believe) school at Laleham under Mr. Buckland, brother of Dean Buckland. Here, we read, "the discipline was of a severity unknown at the present day." Thence he was removed to Eton, where however his stay was short. The poet writes, "the child is father of the man"; but science in those days did not hold the place it does now in the scholastic curriculum, and so the future President, venturing to make some researches into the effects produced by the combination of various detonants, came into collision with the "powers that be";[2] the upshot of this *contretemps* was that the brothers Spottiswoode were transferred to Harrow, then under the rule of the present Bishop of Lincoln. His house tutor was Mr. Harris, of the Park.[3] On entrance he was placed in the upper shell, a high form in those days for a newcomer: here he was a very studious, quiet, and thoughtful boy, not much given to athletic games. He remained at Harrow three years, and in 1842 obtained a Lyon Scholarship.[4] In this same year he entered Balliol College, Oxford, and had the present Bishop of Exeter for his mathematical tutor; subsequently, in 1845, the last year of his residence as an undergraduate, he read with the Rev. Bartholomew Price, of Pembroke College. This gentleman writes: "He showed extraordinary liking for, and great skill in, what I may call the morphology of mathematics, such as the theory of simultaneous equations and the results deducible from the *form* of these equations, a department in which he has since shown great ability. He had, I think, greater taste for these branches in their algebraical and geometrical developments than for any other. His power of work was very great and his industry equally so; he read a great deal outside the usual range." In 1845 he took a first class in mathematics, and he afterwards won the Junior (1846) and Senior (1847) University Mathematical Scholarships. He returned to Oxford for a term or two, and gave a course of lectures in Balliol College on Geometry of Three Dimensions—a favourite subject of his. He was Examiner in the Mathematical Schools in 1857-58.[5] On leaving Oxford, he immediately, we believe, took an active part in the working management of the business of the Queen's printers, about this time resigned to him by his father, Andrew Spottiswoode, brother of the Laird of Spottiswoode. The business has largely developed under his hands.

[1] John Spottiswoode, born 1565, Archbishop of St. Andrews, "had few equals, and was excelled by none"; another John (1616) was "a youth of extraordinary parts"; and Sir Robert Spottiswoode, second son of the Archbishop, was "a man of extraordinary parts, learning, and merit." ("Genealogy of the Spotswood Family in Scotland and Virginia," by C. Campbell. Albany, 1868.)

[2] "The feeling and opinion" at Harrow "were that no blame whatever attached to them."

[3] One who knew Mr. Spottiswoode in his earliest days says: "Our numbers at the school were comparatively very small, but I remember well the great ease with which he did all his school work. I knew him well at Oxford, and he several times lent me his horse—a sturdy, Roman-nosed animal of great courage and strength—for a day's hunting. He rode but little himself, and did not read much in an orderly way." He also gives other particulars of interest, which we forbear to give here.

[4] A son of Bishop Colenso also obtained a scholarship in the same year. The mathematical prizes of the present day were not then founded, so that the name of Spottiswoode does not occur among the prizemen of that time.

[5] He also acted as an Examiner in the Civil Service Commission in its first year of operation, and subsequently for the Society of Arts, and also for the Cowper Street Middle-Class Schools.

Other subjects than mathematics have occupied his attention: at an early age he studied languages, as well Oriental as European; of his acquaintance with these ample evidence is furnished by his contributions more particularly referred to below.

In 1856 Mr. Spottiswoode made a journey through Eastern Russia; of this he has published a graphic and, in parts, very lively account in his book entitled "A Tarantasse Journey through Eastern Russia in the Autumn of 1856" (Longmans, 1857). "I neither made the journey, nor do I now write, with any political object, but simply as a traveller to whom every square mile of the earth's surface is interesting, and the more so in proportion as it is less known."[1]

In 1860 the brothers Spottiswoode, accompanied by a sister, went through Croatia and Hungary.[2] In 1861 Mr. Spottiswoode married the eldest daughter of the late William Urquhart Arbuthnot, a distinguished member of the Indian Council. His exceptional qualifications as an organiser have not only served to advance his business in the way we have mentioned above, but these same qualifications, together with the broad and liberal education on which they were based, have combined to raise him to his present high position in science. As Treasurer and President he has been continuously on the Council of the Royal Society for a great many years, and through his exceptional gifts as an administrator he has rendered it invaluable services. He has rendered similar services to the British Association, to the London Mathematical Society, and to the Royal Institution.[3] We have permission to make the following extract from a letter written by a friend of many years standing: "In the councils (of the various societies) he has always been distinguished by his sound judgment and his deep sympathy with their purest and highest aims. There never was a trace of partisanship in his action, or of narrowness in his sympathies. On the contrary, every one engaged in thoroughly scientific work has felt that he had a warm supporter in Spottiswoode, on whose opportune aid he might surely count. The same breadth of sympathy and generosity of sentiment has marked also his relations to those more entirely dependent upon him. The workmen in his large establishment all feel that they have in him a true and trustworthy friend. He has always identified himself with their educational and social well-being."[4] We give here a list of some of the offices Mr. Spottiswoode has held, and of the honours that have been bestowed upon him: Treasurer of the British Association from 1861 to 1874, of the Royal Institution from 1865 to 1873, and of the Royal Society from 1871 to 1878. In 1871 he succeeded Dr. Bence

[1] The hotel accommodation was of the scantiest (p. 23); the description of the vehicles is pleasanter to read than to realise. The only peculiarly personal statement is that the writer was a non-smoker. There are several illustrations by the author, and a route map of Russia.

[2] For a description, see a paper by Mr. G. A. Spottiswoode in Galton's "Vacation Tourist in 1860."

[3] He has, we believe, also rendered valuable services to the Astronomical and Geographical Societies.

[4] This last statement we have corroborated from other sources. "Spottiswoode's people" have "many institutions for healthful recreation as well as mental improvement, such as library, rowing and cricket clubs, a choral society, and a volunteer corps."

Jones as Honorary Secretary to the Royal Institution. *President* of Section A, 1865; of the British Association, 1878; of the London Mathematical Society, 1870 to 1872; of the Royal Society, 1879, which office he still holds. Correspondent of the Institut (Académie des Sciences), March 27, 1876. He is also LL.D. of the Universities of Cambridge, Dublin, and Edinburgh, D.C.L. of Oxford, and F.R.A.S., F.R.G.S., F.R.S.E. In addition to these honours he has many other literary and scientific distinctions.

Of Mr. Spottiswoode's willingness to communicate from his stores of knowledge many have had frequent proof. We are breaking no faith, we believe, when we mention that it was his wish to purchase the late Prof. De Morgan's valuable library to present it to the Mathematical Society, of which that distinguished mathematician had been the first President.

Few students of the present day are acquainted with Mr. Spottiswoode's earliest work which appeared in the shape of five quarto pamphlets (136 pp. in all) with the title, "Meditationes Analyticæ" (London, 1847). The author's dedication runs thus: "To those who love to wander on the shore till the day when their eyes shall be opened and they shall see clearly the works of God in the unfathomed ocean of truth, these papers are inscribed;" and in his preface he says, "The following papers have been written at various periods, as the subjects presented themselves to notice from time to time. If leisure had been afforded, an attempt would have been made to draw some of them up into a distinct treatise; but it was thought that even in their present form they might interest some of those who take pleasure in the pursuit of mathematical science. Some of the papers are entirely original." The papers are entitled, "Symmetrical Investigations of Formulæ relative to Plane Triangles," "On some Theorems relative to Sections of Surfaces of the Second Order," "On the Reduction of the General Equation of the Second Order," "On the Partial Differential Equations of certain Classes of Surfaces," "On some Theorems relating to the Curvature of Surfaces," "On certain Formulæ for the Transformation of Coordinates," "On the Principle of Virtual Velocities," "On Infinitesimal Analysis," "Examples of the Application of the Infinitesimal Calculus," "On certain Formulæ made use of in Physical Astronomy," "On the Calculus of Variations," "Problems in the Calculus of Variations," and "Note on Lagrange's Condition for Maxima and Minima of Two Variables"—a fair epitome of his subsequent mathematical labours. The treatment calls for no special comment, except that we may note that "in the form of the equations symmetry has been preserved wherever the circumstances of the case would permit."

At a slightly later date (1851) appeared, of a uniform appearance with the "Meditationes," a much more notable pamphlet (63 and viii. pp.), "Elementary Theorems relating to Determinants," of which a writer remarks, "full of interest for the mathematician, but terrible to the unmathematical vision." A second edition of this, rewritten and much enlarged, was published in *Crelle's Journal* (vol. li. 1856, occupying pp. 209–271, 328–381).[1] This was the earliest elementary treatise on a subject which has since risen to such importance, and contains a good sketch of what had previously been done in the same direction. The friend, some of whose words we have already cited, remarks, "that Spottiswoode should have devoted himself at an early period to its cultivation is to me perfectly natural, for the prevailing character of all his mathematical work is *symmetry* (one might generalise still further indeed and say that it, combined with graceful elegance, is the salient feature of all his activity, mathematical, physical, and literary). Bertrand once said of Serret that he was '*un artiste en formules*,' and in a far more general sense one might say that Spottiswoode is the 'incarnation of symmetry.'" To go back to the criticism just now quoted, Mr. Spottiswoode is indeed a leviathan in symbols, and he takes his pastime amongst them: the "gay determinant" is a familiar form nowadays, and "Hamilton's weird delta turned" (the *Nabla* of Clerk Maxwell) is conspicuous on many a page devoted to physics, but in some of the papers we are about to describe there are not only inverted deltas, but Nablas turned to the right and to the left run riot on the pages.

It is since 1870 that Mr. Spottiswoode has more especially divided his attention between physics and mathematics. "His nearest friends," we are informed, "induced him to take up the less abstract one of these two branches of science in order that the general public might have better opportunities of appreciating his abilities. His work in the new field has been of the same character as in the former one. It aims less perhaps at exhaustive treatment than at a study of subtle and beautiful phenomena."

An early consequence of his new study was the publication in 1874, in the NATURE Series, of his "Polarisation of Light." This contains a popular exposition of the subject, and its pages "constitute a talk" with his workpeople "rather than a treatise" on "this beautiful branch of optics."[1]

Before we give a list of the several papers which, of course, do not admit of quotation and passing over, as still within the recollection of most of our readers, the most admirable address delivered before the British Association at Dublin in 1878[2]—though one finds it hard to pass over the many brilliant passages, of more special interest however to the mathematician, who alone can be supposed to care for any other than the ordinary space of three dimensions—we must trespass to the extent of taking the following passage from the earlier address to Section A in 1865. This address, in the words of Prof. Sylvester, is a combined history of the progress of mathematics and physics, and of it Clerk Maxwell said he had endeavoured to follow Mr. Spottiswoode, "as with far-reaching vision he distinguishes the systems of science into which phenomena, our knowledge of which is still in the nebulous stage, are growing."

"A detailed summary of recent progress in pure mathematics would probably prove either interesting to the mathematician or unintelligible to the general hearer;

[1] "On the request of the editor of this *Journal* to reproduce it he (Mr. Spottiswoode) requested permission to revise the work. The subject had, however, been so extensively developed in the interim, that it proved necessary not merely to revise but entirely to rewrite the work. The result is given in the following pages."

[1] He has also contributed a lecture on the same subject to the "Science Lectures at South Kensington" Series.

[2] See NATURE, vol. xviii. pp. 404-415.

with a view to sparing the patience of both, I shall restrict myself to a few general remarks. In both the great branches of mathematics, viz. geometry and algebra, new schools have arisen within the last few years. In its primary aspect the movement has tended to separate the two; geometry has become more purely geometrical in its conceptions and methods, algebra more independent of geometrical considerations. The geometry of to-day is more like the Greek than was that of fifty years ago; and yet at the same time they have not only many principles really in common, but many methods which, although independent, are strictly analogous. Geometry regards its figures, algebra its forms, not as isolated individuals, but as associated with others (concomitants, as they are called) whose properties characterise those of their primitives. The principles of both may be regarded as the same, but dual in their application. Geometry, again, is dual within itself: points and lines may be so viewed that theorems concerning the one give rise to analogies concerning the other; the principle the same, but dual in its manifestation. In this way we seem to be rising to laws which transcend the distinctions between the two parts of geometry—between geometry and algebra.

"Descending a little further into particulars, in another way again we seem to be gaining some steps—but as yet only a few steps—towards a higher scheme both of geometry and algebra. There are a few certain relations so elementary in their conception, yet so universal in their application, that they seem capable of forming the basis of extensive theories: such, for example, in geometry, is that of Anharmonic Ratio—a particular kind of ratio applicable alike to points and rays, to lines and to angles, on which M. Chasles has founded his new and classical work on Conic Sections. Such, again, in algebra, are those of homogeneity and of symmetry, which prove to be not merely improvements in form, but actually new powers for progress in the hands of the mathematician. The calculus of homogeneous forms has marked a new era in the history of algebra; the theory of equations has been transfigured in its light; mechanics, both ordinary and molecular, have been elucidated by it; and the remote applications of the integral calculus have felt its ever-extending influence. Under these, as it were, new fundamental conceptions, whole theories may be coordinated, and of these, again, perhaps some coordination may one day be contemplated. As another instance of this generalisation of principles and of this dual aspect of the principles so generalised within almost the present generation, it has been discovered, or at all events been duly realised, that symbols of operation combine according to definite laws, comprising as a particular case those of ordinary number. This fertile idea has, year by year, been receiving fuller developments, till it has at last assumed the form of a complete calculus."

We, too, must join our apologies with those of the learned speaker for lingering so long upon a favourite subject.

The following is as complete a list of Mr. Spottiswoode's papers as we have been able to make:[1] they are grouped, not according to subjects nor in order of time, but as they occur in the several journals in which they originally appeared:—

Phil. Magazine.—(1) On the Equation $Q=q\ (\omega, x, y, z) = w + ix + jy + kz$ (vol. xxxvi. 1850); this is a theorem of considerable importance in the calculus of quaternions, and indeed essential for the application of that method to geometrical and physical problems. (2) On the Quaternion Expressions of Coplanarity and Homoconicism (*ib.*). (3) On the Geometrical Interpretation of Quaternions (vol. xxxvii. 1850), the working out on other lines of results stated in a previous volume by Prof. Donkin. (31) On a Geometrical Theorem (*ib.*, 1850), viz. if three cones of the second order, having a common vortex, intersect one another two and two, the nine lines of intersection (three being selected from each pair of cones) will lie on a cone of the fourth order. (7) On a Problem in Combinational Analysis (vol. iii. 1852) connected with the 15-girl Problem and a more general form of it, the solution of which turns upon certain determinants. (41) On some Experiments on Successive Polarisation of Light made by Sir C. Wheatstone (vol. xli. 1871); the introduction of instrumental means for converting the plane of polarisation of the ordinary apparatus into successive, or, as it is more commonly called, circular polarisation, and the explanation of the phenomena thence arising, constitutes the main purpose of the communication. See also *Proc. of R. Inst.*, vol. vi. 1872. 1875 (*a*) on a Revolving Polariscope; 1882 (*b*) on a Separator and Shunt for Alternate Currents.

Camb. and Dub. Math. Journal.—(4) On certain Geometrical Theorems (vol. vi. 1851). This is an anonymous article which gives simple algebraical demonstrations of certain of Steiner's Theorems in the *Systematische Entwickelung*, and also of some relations given by M. Chasles in his "Aperçu." (9) On Certain Theorems in the Calculus of Operations (vol. viii. 1853); an extension of theorems by Boole (*Phil. Trans.*, 1844), relating to the operation symbol $D = x\frac{d}{dx}$, and by Carmichael relating to the symbol $\nabla = x_1\frac{d}{dx_1} + x_2\frac{d}{dx_2}\ \ldots$ to the cases (1) in which the order of the Variables by which the Symbols of Differentiation are Multiplied is not the same as that of the Variables with respect to which the Differentiations are to be performed; (2) in which the Variables by which the Symbols of Differentiation are Multiplied are any linear Function of the Given Variables; (10) on Certain Geometrical Theorems (*ib.* 1853); two Elementary Theorems in anharmonics proved by aid of determinants. (11) On the Curvature of Curves in Space (vol. ix. 1854); on this M. Chasles ("Rapports," p. 162) remarks: "M. W. Spottiswoode est parvenu à la même expression dans une Note . . ." *i.e.* to the expression—
$$\frac{1}{\rho} = \operatorname{cosec}\phi\left\{\frac{1}{\rho^2} + \frac{1}{\rho_1^{\,2}} - \frac{2\cos\phi}{\rho\rho_1}\right\}^{\frac{1}{2}}.$$

Quarterly Journal of Mathematics.—(15) Note on Axes of Equilibrium (vol. i. 1857). The axes (Möbius, "Statik") possess the property of allowing the body to be turned about them, the forces retaining their directions in space without a disturbance of equilibrium. The paper is an application of formulæ given by Rodrigues to a proof of the property. (16) On a Theorem in Statics (*ib.* 1857) is a proof of the following, due to Möbius ("Statik"): "If there be any forces in equilibrium, and a series of pyramids be constructed having for one edge a common line, and for their opposite edges the lines which represent the forces, in both magnitude and direction, respectively, the algebraical sum of the volumes of the pyramids will vanish. It is of this M. Chasles ("Rapport," p. 59) writes: "M. W. Spottiswoode, à qui toutes les ressources des nouvelles théories de l'analyse sont si familières, s'est plu à les appliquer à la démonstration de cette proposition (*i.e.* Möbius's) et d'un autre passage du traité de statique de Möbius, sur les axes de l'équilibre." (23) On Petzval's Asymptotic Method of Solving Differential Equations (vol. v. 1862). Also in a somewhat different form in *Brit. Assoc. Report* (part ii.), 1861. (29)

[1] We trust our readers will pardon our imperfect treatment of these papers: we had formed quite a mass of notes—a "rudis indigestaque moles"—but we have had, through circumstances over which we had no control, an utterly inadequate period in which to prune them and shape them into comely form. The prefixed numbers are those of the "Royal Society's Catalogue" and the notes are in most cases derived from the papers themselves. In our haste we have preferred to insert notes to the less familiar papers; the papers read before the Royal and Mathematical Societies are without doubt those by which Mr. Spottiswoode's rank as a mathematician has been determined, but these are just the ones that are most familiar to students.

On Differential Resolvents (vol. vi. 1863). A subject first brought into notice by Mr. J. Cockle, subsequently discussed by Rev. R. Harley. The functions considered are derived from equations in a factorial form; see also *Manch. Phil. Soc. Memoirs*, ii. 1865. [In the *R. S. Catalogue* these are also numbered (33)]. (37) Note on the Contact of Curves (vol. vii. 1866). "In my former paper" (*Phil. Trans.* 1862, see *infra*) "one set of expressions is unsymmetrical with respect to the variables; the other, although symmetrical, involves certain arbitrary quantities which remain to be eliminated by special methods in the course of the developments"—the object of the note is to establish general expressions which are both symmetrical and free from arbitrary quantities. (38) Note on the Resolution of a Ternary Cubic into Linear Factors (*ib.* 1866) is in effect a note on a paper by Mr. J. J. Walker in the previous volume, entitled "On the Resolution of Composite Quantities into Linear Factors."

Crelle's Journal.—(5) Mémoires sur les points singuliers d'une courbe à double courbure (vol. xlii. 1852). (6) Mémoire sur quelques formules relatives aux surfaces du second ordre (*ib.* 1852). (12) Correspondence between Prof. Donkin and Mr. Spottiswoode (vol. xlvii. 1854); extracts from letters (one from each) on a Method for Determining Two Cyclic Sections of a Surface of the Second Order. (14) The Memoir on Determinants (vol li. 1856). (25) Sur quelques formules générales dans le calcul des opérations (vol. lix. 1861), connected with a *Phil. Trans.* paper (17). In this he shows the method by which he obtained the formulæ in (17). (32) Note sur la transformation de la cubique temaire en sa forme canonique (vol. lxiii. 1864).

Tortolini Annali di Scienze.—(8) Sulla trasformazione delle equazioni differenziali lineari dell' ordine secondo (vol. iii. 1852).

R. Soc. Proc.[1]—(13) Researches on the Theory of Invariants (vol. vii. 1854). "The view of invariants here taken has suggested a series of other functions of which invariants form the last term. These functions I propose to call *variants*. With the exact relation between these functions and covariants I am not at present acquainted." (17) On an Extended Form of the Index Symbol in the Calculus of Operations (vol. x. 1859, *Phil. Trans.* 1860). A more detailed form of (9). (20) On the Calculus of Functions (vol. xi. 1861). (21) On Internal and External Division in the Calculus of Symbols (*ib.*). Connected with a paper by Mr. W. H. L. Russell (*Phil. Trans.* 1861), a generalisation and an extension. (30) On the Equations of Rotation of a Solid Body about a Fixed Point (vol. xiii. 1863). In treating the equations of rotation of a solid body about a fixed point it is usual to employ principal axes of the body as the moving system of coordinates. Cases, however, occur in which it is advisable to employ other systems. The object of the paper is to develop the fundamental formulæ of transformation and integration for any system. [This is also given as (34) in the *R. S. Cat.*]. (35) On the Sextactic Points of a Plane Curve (vol. xiv. 1865; *Phil. Trans.* 1865). (40) On the Contact of Conics with Surfaces (vol. xviii. 1870; *Phil. Trans.* 1870). (43) On the Contact of Surfaces (vol. xx. 1872; *Phil. Trans.* 1872). (45) On the Rings Produced by Crystals when Submitted to Circularly Polarised Light (vol. xx. 1872); 1874 (*a*) On Combinations of Colour by Polarised Light; 1874-5 (*b*) On Stratified Discharges through Rarefied Gases; 1875-6 (*c*) On Multiple Contact of Surfaces; (*d*) An Experiment in Electromagnetic Rotation; 1876-7 (*e*) On Stratified Discharges (ii.); Observations with a Revolving Mirror (iii.); (*f*) On a Rapid Contact Breaker and the Phenomena of the Flow; 1877 (*g*) On Hyperjacobian Surfaces and Curves; (*h*) Stratified Discharges (iv.): Stratified and Unstratified Forms of the Jar-Discharge; (*i*) Photographic Image of the Stratified Discharge; 1878 (*j*) Stratified Discharge (v.); Discharge from a Condenser of Large Capacity; 1879 (*k*) On the Sensitive State of Electrical Discharges through Rarefied Gases [with J. F. Moulton], *Phil. Trans.;* 1879-80 (*l*) On some of the Effects Produced by an Induction Coil with a De Meriten's Magneto-Electric Machine; (*m*) On the Sensitive State (ii.) (with J. F. M.), *Phil. Trans.;* 1880-1 (*n*) On the 48 Coordinates of a Cubic Curve in Space (*Phil. Trans.*); 1881 (*o*) On Stratified Discharges (vi.), Shadows of Striæ (with J. F. M.); and (*p*) Multiple Radiations from Negative Terminal; 1881-2 (*q*) Note on Mr. Russell's Paper on Definite Integrals; (*r*) Note on Mr. Russell's Paper on Certain Geometrical Theorems; (*s*) On the Movement of Gas in Vacuum Discharges (with J. F. M.).[1]

R. Asiatic Soc. Journal.—(18) Note on the supposed Discovery of the Principle of the Differential Calculus by an Indian Astronomer (vol. xvii. 1860). While not granting that Bhóskarácharya had discovered the principle, "it must be admitted that the penetration shown by him in his analysis is in the highest degree remarkable, and that the formula which he establishes and his method of establishing it bear more than a mere resemblance—they bear a strong analogy—to the corresponding process in modern astronomy." (28) On the "Súrya Siddhánta" and the Hindoo Method of calculating Eclipses (vol. xx. 1863). It had been suggested that Mr. Spottiswoode should undertake an edition of the above work. For reasons stated, the attempt was not made; but the object of this paper is the translation into modern mathematical language and formulæ of the rules of the work in question.

R. Geog. Soc. Proc.—(19) On Typical Mountain Ranges: an application of the Calculus of Probability to Physical Geography (vol. iv. 1861; *Journal*, vol. xxxi. 1861).

R. Astron. Soc. Memoirs.—(22) On a Method for determining Longitude by Means of Observations on the Moon's greatest Altitude (vol. xxix. 1861; also in *Geog. Soc. Proc.* vol. v. 1861).

British Assoc. Report.—(24) On the Reduction of the Decadic Binary Quantic to its Canonical Form (1861, part 2); (36) Address to Section A (1865); (*a*) Address to the Association (1878).

Phil. Trans.—(26) On the Contact of Curves (1862); (27) On the Calculus of Symbols (1862); 1874 (*a*) On the Contact of Quadrics with other Surfaces. See also above under *R. Soc. Proc.*

Comptes Rendus.—(39) Note sur l'équilibre des forces dans l'espace (vol. lxvi. 1868); (48) Note sur la représentation algébrique des lignes droites dans l'espace (vol. lxxvi. 1873); (49) Sur les plans tangents triples à une surface (vol. lxxvii. 1873); 1874 (*a*) Sur les surfaces osculatrices; 1875 (*b*) Sur la représentation des figures de géométrie à *n* dimensions par les figures corrélatives de géométrie ordinaire; 1876 (*c*) Sur le contact d'une courbe avec un faisceau de courbes doublement infini.

R. Inst. Proc.—(44) On Optical Phenomena produced by Crystals submitted to Circularly Polarised Light (vol. vii. 1872. See also *Phil. Mag.* vol. xliv. 1872); (46) On the Old and New Laboratories at the Royal Institution (vol. vii. 1873); (47) On Spectra of Polarised Light (*ib.* 1873); 1874 (*a*) On Combinations of Colour by Polarised Light; 1878 (*b*) A Nocturne in Black and Yellow; (*c*) Quartz: an old chapter rewritten; 1880 (*d*) Electricity in Transitu; 1882 (*e*) Matter and Magnetoelectric Action.

Musical Society Proc.—1879 (*a*) Lecture on Beats and Combination Tones.

Royal Society.—Presidential Addresses for the Years 1879, 1880, 1881, 1882.

[1] When there is a paper in the *Phil. Trans.* as well, the reference is also given under this head.

[1] For analyses of the papers on "Sensitive Discharges," &c., consult vol. ii. of "A Physical Treatise on Electricity and Magnetism," by J. E. H. Gordon, 1880 (see pp. 47-50, 71-81, 88-111).

L. Math. Soc. Proc.—1866 (*a*) A Problem in Probabilities connected with Parliamentary Elections; 1868 (*b*) Equilibrium of Forces in Space; 1871 (*c*) Question in the Mathematical Theory of Vibrating Strings; 1872 (*d*) On some recent Generalisations in Algebra (Presidential Address); 1874 (*e*) On the Contact of Quadrics with other Surfaces; 1876 (*f*) On Determinants of Alternate Numbers; (*g*) On Curves having Four-point Contact with a Triply-infinite Pencil of Curves; 1879 (*h*) On the Twenty-one Coordinates of a Conic in Space; (*i*) On Clifford's Graphs; 1881 (*j*) On the Polar Planes of Four Quadrics; 1882 (*k*) On Quartic Curves in Space.

"*A MANUAL OF THE INFUSORIA*"

A Manual of the Infusoria; Including a Description of all known Flagellate, Ciliate, and Tentaculiferous Protozoa. By W. Saville Kent, F.L.S., F.Z.S. (London: David Bogue, 1882.)

THE *Philosophical Transactions of the Royal Society of London* for the year 1677 contain the first published account of the minute organisms to which the term "Infusoria" is now very generally applied. The account is by "Mr. Antony van Leeuwenhoek," who, taking up the line of study so successfully pursued by his compatriot, Swammerdam, was the first to apply the microscope to the investigation of the otherwise invisible fauna and flora which teem in inconceivable abundance in the waters of ponds, rivers, and seas, in the infusions of organic substances prepared by man's agency, and in even the minutest drops of moisture which accumulate on the surfaces of natural objects.

Henry Baker (1742), O. F. Müller (1773), and other names are connected with the history of this field of investigation in the period antecedent to Ehrenberg, who in 1836 gave a new aspect to the subject by his descriptions and figures of a great number of forms and of their intimate organisation. The minute creatures at one time spoken of as "animalculæ," and later as "Infusoria," are now known to comprise many very diverse series of organisms—unicellular plants, variously organised unicellular animals, as well as animals of multicellular structure and high organisation, although of minute size. The improvement of the microscope within the last forty years and the studies of a host of observers, among whom are Dujardin (1841), von Siebold (1845), Stein (1854), Claparède and Lachmann (1858), Max Schultze (1860), and more recently of Haeckel, Engelmann, and Bütschli—have gradually resulted in the recognition of a series of minute animals included amongst the "animalculæ" and "Infusoria" of earlier writers, which are characterised by having their living substance in the form of one single nucleated corpuscle or "cell," whilst nevertheless exhibiting a considerable degree of organisation, possessing a mouth into which solid particles of food are taken, pulsating spaces within the protoplasm of the cell, special organs of locomotion, prehension, and protection. These are the mouth-bearing Protozoa, distinguished as such from the other unicellular animals which have not a special orifice for the ingestion of food and constitute the mouthless Protozoa.

It is to these mouth-bearing Protozoa and a few allied mouthless forms that Mr. Saville Kent restricts (as is not unusual) the old term Infusoria. Among them the most numerous and highly organised are the Ciliata; far less abundant and varied are the Tentaculifera (Acinetæ), whilst the Flagellata have, on account of their excessive minuteness, not been properly understood (and were for the most part altogether unknown) until very recently, some important features in their organisation having been first made known by the author of the book which forms the text of this article.

Mr. Kent's "Manual of the Infusoria" consists of two large volumes and an atlas of fifty-one plates. The first volume contains chapters on the history, the organisation, the affinities, and the classification of the Infusoria. Then the three classes, Flagellata, Ciliata, and Tentaculifera, are taken up one by one and systematically divided into orders and families, genera and species—a diagnosis and usually a figure being given of every species. The systematic treatment of the Ciliata and Tentaculifera occupies the second volume. Altogether Mr. Kent describes 900 species of Infusoria, arranged in 300 genera and 80 families. To go over this ground in any case involves a vast amount of labour and perseverance. To do so in the thorough and conscientious manner which distinguishes Mr. Kent's work requires special capacity. Mr. Kent has spared no pains to make his work a trustworthy source of information on all points relating to the group with which it deals; the most comprehensive as well as the smallest and most obscure of recent publications relating to the organisation or to particular species of Infusoria have their contents duly set forth in the proper place in Mr. Kent's work. So far as a frequent reference to these volumes enables one to come to a conclusion, little if anything of importance, whether published in English, French, German, or Italian, has been overlooked by our author. Even the quite recent observations of Foettinger on the parasitic *Benedenia* found in Cephalopoda, and of Joseph Leidy on the parasitic Ciliata occurring in the Termites, are incorporated, as well as the observations of Cunningham on *Protomyxomyces*, little more than a year old.

This is by no means the only merit of Mr. Kent's work. He might have contented himself with recasting the materials to be found in the three great volumes published by Stein, in Claparède and Lachmann, and in Pritchard's "Infusoria" (a valuable book in its day), and have simply incorporated with these the results scattered through the various English and foreign journals and transactions of the past twenty-five years. Mr. Kent has duly done all this, but he has done more, since he has himself made a very careful and prolonged study of a large number of Infusoria. Accordingly we find throughout the present work original observations brought forward for the first time. These include a number of new species and genera, especially among the Flagellate and Tentaculiferous forms. The beautiful cup-forming monads mounted on branching stalks like a colony of Vorticellæ were first brought prominently into notice by that keen observer, the late Prof. James-Clark of Boston, and Mr. Kent has followed up the study of these beautiful forms in a very thorough manner. On the whole, it may be said that the portion of Mr. Kent's work devoted to the Flagellata will have, for those naturalists who have not very closely followed the periodical literature of the subject, the charm of complete novelty, for very many of these forms were completely unknown or misunderstood